

Give the complete definition of the definite integral as presented in lecture.

SCORE: \_\_\_\_\_ / 4 PTS

SEE HANDOUT ON MY WEBSITE

Use the **definition** of the definite integral and the theorem about continuous integrands to evaluate  $\int_{-3}^1 (2x - 4) dx$ . SCORE: \_\_\_\_ / 9 PTS

**NOTE: 0 points if you use the Fundamental Theorem of Calculus or geometry instead.**

REMEMBER THIS  
OVERRIDING RULE

$$\begin{aligned} & \lim_{n \rightarrow \infty} \sum_{i=1}^n f(a + i\Delta x) \Delta x \\ \textcircled{1} &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(-3 + \frac{4i}{n}\right) \frac{4}{n} \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n \left[2\left(-3 + \frac{4i}{n}\right) - 4\right] \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n \left(-10 + \frac{8i}{n}\right) \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \left[ \sum_{i=1}^n -10 + \frac{8}{n} \sum_{i=1}^n i \right] \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \left[ -10n + \frac{8}{n} \frac{n(n+1)}{2} \right] \\ &= \lim_{n \rightarrow \infty} 4 \left[ -10 + \frac{4(n+1)}{n} \right] \\ &= 4(-10 + 4) \\ &= -24 \end{aligned}$$

The graph of function  $f$  is shown on the right.

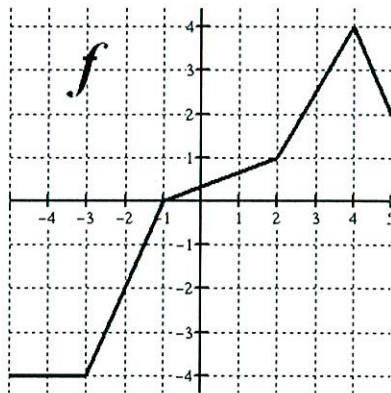
SCORE: \_\_\_\_ / 7 PTS

[a] Find  $\int_{-2}^4 f(t) dt$ .

$$\begin{aligned} &= \int_{-2}^{-1} f(t) dt + \int_{-1}^2 f(t) dt + \int_2^4 f(t) dt \\ &= \underbrace{-\frac{1}{2}(2)(1)}_{\textcircled{2}} + \underbrace{\frac{1}{2}(1)(3)}_{\textcircled{1}} + \underbrace{\left(\frac{1+4}{2}\right)2}_{\textcircled{1}} \\ &= -1 + \frac{3}{2} + 5 \\ &= \underbrace{\frac{11}{2}}_{\textcircled{1}} \end{aligned}$$

[b] Find  $\int_5^4 f(t) dt$ .

$$\begin{aligned} &= -\int_4^5 f(t) dt \\ &= -\left(\frac{4+2}{2}\right)1 = \underbrace{-3}_{\textcircled{1}} \end{aligned}$$

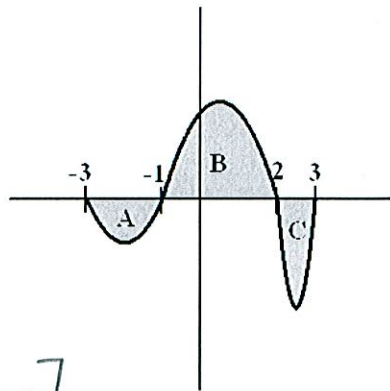


The graph of  $f(x)$  is shown on the right. If the area of shaded region  $A$  is 3,

SCORE: \_\_\_\_\_ / 6 PTS

the area of shaded region  $B$  is 8, and the area of shaded region  $C$  is 2, find  $\int_{-3}^3 (7 - 4f(x)) dx$ .

**For full credit, you must clearly show the use of all necessary properties of the definite integral.**  
**Minimal credit will be given for arithmetic alone.**



$$\begin{aligned} &= \int_{-3}^3 7 dx - \int_{-3}^3 4f(x) dx \textcircled{1} \\ &\textcircled{1} = 7(3 - (-3)) - 4 \int_{-3}^3 f(x) dx \textcircled{1} \\ &= 42 - 4 \left[ \int_{-3}^{-1} f(x) dx + \int_{-1}^2 f(x) dx + \int_2^3 f(x) dx \right] \textcircled{1} \\ &= 42 - 4 [-3 + 8 - 2] \textcircled{1} \\ &= 30 \textcircled{1} \end{aligned}$$

Sketch a region whose area is given by the expression  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2}{n} e^{-5 + \frac{2i}{n}}$ . Label your graph clearly. **SCORE: \_\_\_\_\_ / 4 PTS**

$$\Delta x = \frac{b-a}{n} = \frac{2}{n} \rightarrow b-a=2$$

$$f(a+i\Delta x) = f(a + \frac{2i}{n}) = e^{-5 + \frac{2i}{n}}$$

$$a = -5 \rightarrow b = -3$$

$$f(x) = e^x$$

